

# Aggregating Causal Judgments\*

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## Abstract

Decision making typically requires judgments about causal relations: we need to know both the causal effects of our actions and the causal relevance of various environmental factors. Judgments about the nature and strength of causal relations often differ, even among experts. How to handle such diversity is the topic of this paper. First, we consider the possibility of aggregating causal judgments via the aggregation of probabilistic ones. The broadly negative outcome of this investigation leads us to look at aggregating causal judgments independently of probabilistic ones. We do so by transcribing causal claims into the formal judgment-aggregation framework and applying some recent results in this field. Finally, we look at the implications for probability aggregation when it is constrained by prior aggregation of qualitative causal judgments.

## 1 Introduction

Decision making typically requires judgments about causal relations. Home owners need to know whether putting locks in their doors will make their houses more secure. Jurors need to know whether the accused is causally responsible for damages before they can assess whether they are legally responsible. Aid agencies need to know how the different projects they can invest in will affect the lives of those they are concerned about; and so on. Opinions about the nature and strength of causal relations often differ, even among experts. How to handle such diversity of opinion is the topic of this paper. We investigate the possibility of coherently aggregating different causal judgments into a single one that may be applied to the decision problem at hand.

The basic set-up of this aggregation problem is as follows. Individuals hold particular judgments about both the nature of the causal relations between the variables

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in some set  $\mathbf{V} = \{V, W, \dots\}$  and the probabilities of these variables taking certain values, unconditionally or conditionally on the values of other variables. The task is to construct a single aggregate judgment on the causal relations between the variables and the relevant probabilities in a way that preserves, as much as possible, the information contained in the individuals' judgments. For present purposes, we assume that individuals' judgments are coherent. More generally, one might allow (localized) incoherence in some of the individuals' judgments, or allow that individuals do not make judgments about all causal relations or all probabilities in question. Their judgments could be restricted to just certain variables relevant to the decision problem at hand, or further still, to just some subset of them or just one type of judgment: causal or probabilistic.

The causal judgments of individuals could be represented in a number of different ways, but here we adopt the Bayesian network framework familiar from the work of Pearl [16], Spirtes, Glymour and Scheines [17] and others in which they are represented by directed acyclic graphs (DAGs) and associated conditional probabilities. We do not intend thereby to take up a position on the nature of causal judgments, nor on the question of whether they can ultimately be analysed probabilistically.<sup>1</sup> Anyone who holds the view that causal judgments are just features of probability judgments – for instance, that to judge that  $X$  causes  $Y$  is to hold certain conditional probabilistic judgments including that the conditional probability of  $X$  given  $Y$  exceeds its unconditional probability – is free to regard the DAG representations of an individual's causal judgment as adding no information to his or her probability judgments. We also do not think that much of what we claim depends on the DAG representation; we could just as well represent causal judgments by, for instance, counterfactual beliefs of the right kind.

A DAG represents an individual's *qualitative* judgment of causal relevance and irrelevance between variables. Her *quantitative* judgment of causal dependence is reflected in the associated conditional probabilities for the values of these variables, given the values of any variables on which they are directly causally dependent. Then the individual's unconditional probabilities for the values of the given variables can be computed from their conditional probabilities together with the individual's unconditional probabilities for the parent variables. Consider the following example, which we will use at various points in the discussion.

**Example: Predicting famine.** An aid agency wishes to do some advance planning for its famine relief operations and consults a number of experts in order to determine the risk of famine in some particular region. All agree that the relevant variables are  $R$ : rainfall,  $Y$ : crop yields,  $P$ : political conflict, and of course  $F$ : famine. But they disagree both on the causal relations between the four variables and on the

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<sup>1</sup>A probabilistic analysis may involve variables not included in the DAGs we consider.

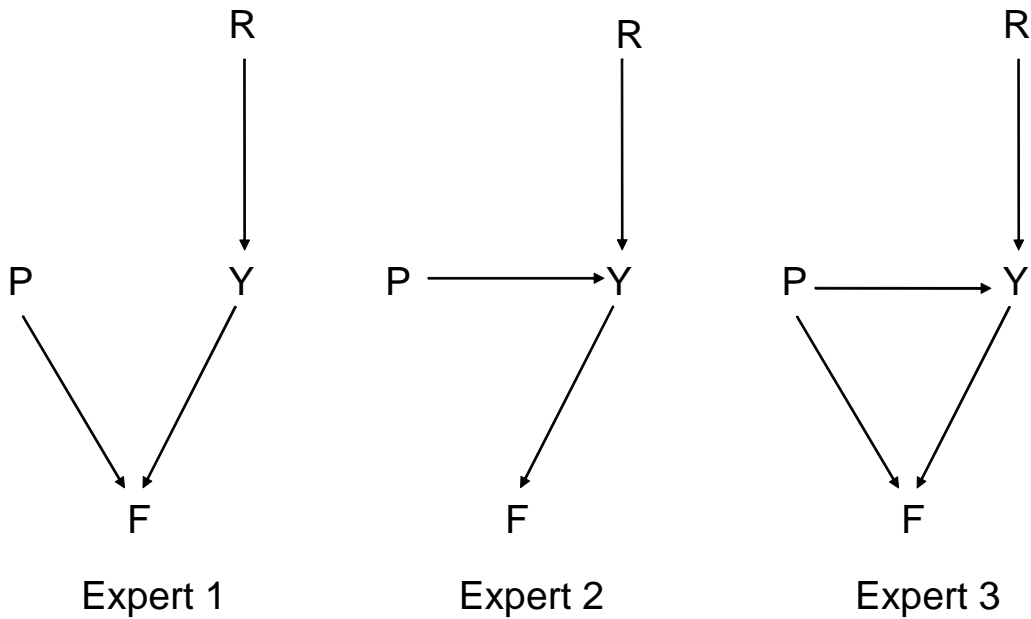


Figure 1: Expert causal judgments

probabilities of the various values the variables may take. All consider rainfall to be the main determinant of crop yield, but while Expert 1 thinks that poor crop yield and disruptive political conflict are the main causes of famine, Expert 2 thinks that the causal influence of political conflict on famine is indirect, via the effect of the disruption of agricultural production on crop yields. Expert 3 considers the relationship between political conflict and famine to be more complicated still, with political conflict both causing famine directly, by disrupting food distribution, and indirectly, through the influence on crop yields. These three opinions are represented in Figure 1 by a set of DAGs.

The fact that individuals make both causal and probabilistic judgments raises the question of whether aggregation of both kinds of judgments should be conducted all at once or in two stages. In Section 2, we focus on what we call *one-stage aggregation*, in which only probability judgments are aggregated. This approach draws on the standard literature on probability aggregation. It is motivated mainly by the thought that the probability judgments of individuals reflect their causal judgments in various ways and hence that the problem of causal judgment aggregation may be solved by constraining probability aggregation so as to preserve the causal information contained in probability judgments. Our verdict on this possibility is broadly negative, however. In Sections 3 to 5, we therefore pursue an alternative *two-stage* approach, aggregating first the qualitative causal judgments represented by the DAGs (Section 3) and then the quantitative probabilistic ones (Sections 4 and 5), on the

assumption that a consensus about the causal relations between variables has been reached.

## 2 One-stage aggregation

The problem of aggregating causal judgments has not received much attention, at least in the form presented here, but there is a vast literature on aggregating expert opinion, mainly in statistics, and especially on aggregating expert probabilities. In this section, we draw on this literature to examine the possibility of reasonable one-stage aggregation of individuals' judgments. One-stage aggregation may be the only method available in cases in which individuals either make no causal judgments or their causal judgments are very incomplete. It is natural, moreover, for those holding a probabilistic view about causation to rely only on this method. But one-stage aggregation may also be motivated by the less controversial thought that the causal judgments of individuals are reflected in (even if they are not reducible to) the relations between the individuals' unconditional and conditional probabilities for the relevant events. If this is so, then even on a non-reductionistic view about causal judgments one may hope that probability aggregation could be constrained in a manner which preserves the causal judgments implicit in probabilistic ones.

Broadly, the probability aggregation literature can be divided into three approaches to the problem: linear pooling, geometric pooling and supra-Bayesian approaches. (A useful guide to this literature can be found in the survey paper of Genest and Zidek [8]; here we draw only on salient aspects.) The last approach is directed at a slightly different problem to ours – namely that of how an individual expert should modify his judgments in the light of the expressed judgments of other experts – and so we can set it aside. The other two approaches assume that the experts' opinions have reached an equilibrium state and that no further modification of their viewpoints will take place before the relevant decision has to be made.

Consider an opinion aggregation problem of the following form. A set of *events* is given (e.g., the event “high political conflict” or “low political conflict and famine”), and the task is to merge the probability judgments of individuals  $1, \dots, n$  (the “experts”) on these events into an aggregate probability judgment on the events.<sup>2</sup> So, we have to merge (individual) probability functions  $\text{Pr}_1, \dots, \text{Pr}_n$  into an (aggregate) probability function  $\text{Pr}$ . Many aggregation rules are imaginable. Formally, a *probability aggregation rule* is a function that assigns to *any* vector  $\langle \text{Pr}_1, \dots, \text{Pr}_n \rangle$  (called a *profile*) of individual probability functions an aggregate probability function  $\text{Pr}$ .

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<sup>2</sup>Events can be identified with subsets of a given set of possible worlds. In many formal results, the set of events considered (i.e., the domain of the individual probability functions  $\text{Pr}_1, \dots, \text{Pr}_n$  and the aggregate probability function  $\text{Pr}$ ) forms an *algebra*: the negation (complement) of any event is also an event, and the disjunction (union) of two events is an event too.

Of the various possible aggregation rules, *linear pooling* stands out for a variety of formal and conceptual reasons (e.g., Lehrer and Wagner [9], Dietrich and List [6]). In particular, the following axiomatic argument can be given. Let us require the aggregation rule to satisfy two seemingly natural conditions:

**Ind** (*Event-wise Independence*) The aggregate probability of any given event  $X$  depends only on the individuals' probabilities of  $X$  (regardless of the individuals' probabilities of other events  $Y$ ).<sup>3</sup>

**ZP** (*Zero Preservation*) The aggregate probability of any given event  $X$  is zero whenever all individuals give  $X$  zero probability.<sup>4</sup>

Applied to the event “famine”, for instance, Zero Preservation implies that famine is assigned an aggregate probability of zero if all individual experts assign a probability of zero to it. Event-wise independence implies that the aggregate probability of famine depends only on the individual experts' beliefs about the probability of famine, not on their beliefs about the probability of a certain level of crop yield, political conflict, etc. (This is not to deny, of course, that individuals form their beliefs regarding famine in the light of their judgments on crop yield, political conflict etc.)

Perhaps surprisingly, the *only* aggregation rules satisfying these two conditions are linear pooling functions: the aggregate probability of any event  $X$  is a (possibly weighted) arithmetic average of the individual probabilities of  $X$ , i.e.,

$$\Pr(X) = w_1\Pr_1(X) + \dots + w_n\Pr_n(X),$$

where the weights  $w_1, \dots, w_n \geq 0$  add up to one and are the same for all events  $X$ .<sup>5</sup> Examples of linear pooling functions are equal-weight averaging ( $w_1 = \dots = w_n = 1/n$ ) and dictatorial aggregation (some individual  $i$  has weight  $w_i = 1$  and all others have weight 0).

The natural interpretation of these weights is in terms of judgmental competence, so that the choice of a particular linear pooling rule is dictated by considerations of the relative expertise of the individuals whose opinions are being sought. In this light, the fact that linear pooling rules assign weights to the opinions of individuals that are independent of the object of these opinions seems quite unsatisfactory, since individuals may be more or less expert on different kinds of issues and it would seem natural to vary the weights on their opinions to reflect this. The aid agency would do well to consult climatologists, agriculturalists and political scientists to

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<sup>3</sup>Formally,  $\Pr(X)$  is a function of  $\Pr_1(X), \dots, \Pr_n(X)$ . This function may be a different one for different events  $X$ .

<sup>4</sup>Formally,  $\Pr(X) = 0$  if  $\Pr_1(X) = \dots = \Pr_n(X) = 0$ .

<sup>5</sup>See McConway [14], drawing on previous results. The result requires that the set of events considered forms an algebra (see footnote 2) and contains at least three events apart from the contradiction (empty set of worlds) and the tautology (set of all worlds). For a more recent generalization, see Dietrich and List [6].

reach a balanced view on the causes of famine, but in doing so it would be reasonable for it to give more weight to the climatologists' probabilities for rainfall than to the political scientists', but more weight to the political scientists' probabilities for political conflict.<sup>6</sup>

Our main concern here, however, is with the question as to whether such linear pooling functions satisfactorily respect the causal knowledge of the individual experts. An individual's causal judgments will be reflected in certain (unconditional or conditional) independencies in his or her probability judgments. For instance, if individual  $i$  believes that events  $X$  and  $Y$  do not causally affect each other but have a common cause, say event  $Z$ , then he or she will take  $X$  and  $Y$  to be probabilistically independent given  $Z$ ,<sup>7</sup> because any probabilistic correlation between  $X$  and  $Y$  is "screened off" by conditionalising on  $Z$ . A minimal condition of respecting causal judgments is that at least *unanimously* held causal judgments be reflected in the aggregate probability function  $\text{Pr}$ . That is,  $\text{Pr}$  should display at least those (conditional) independencies that are supported by unanimous causal judgments. For example, if *all* individuals judge  $X$  and  $Y$  to be causally independent with common cause  $Z$ , then that independence judgment should be reflected in the aggregate probability function  $\text{Pr}$ . This motivates the following condition on probability aggregation:

**IP** (*Independence Preservation*) For any given events  $X, Y, Z$ , if all individuals  $i$  judge  $X$  and  $Y$  to be probabilistically independent given  $Z$ , then this conditional independence also holds under the aggregate probability function.<sup>8</sup>

Note that, by preserving all unanimous probabilistic independencies (conditional or unconditional), we may also preserve independencies that are not grounded in unanimous *causal* judgments. For instance, it may be that all individuals judge  $X$  and  $Y$  to be independent given  $Z$ , but some do so on the grounds of judging that  $X$  indirectly causes  $Y$  through  $Z$ , others on the grounds of judging that  $Y$  indirectly causes  $X$  through  $Z$ , still others on the grounds of judging that  $X, Y$  and  $Z$  are entirely causally disconnected, and so on. Even in this case of causal disagreement, Independence Preservation requires the preservation of the *probabilistic* (conditional) independence. The purely probabilistic informational basis of one-stage probability aggregation simply does not allow us to distinguish between different motivations (causal or other) behind probabilistic independencies. Without explicit causal information, we have to use Independence Preservation to preserve unanimous causal judgments, although this requires us to preserve even those conditional independencies that are not causally motivated.

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<sup>6</sup>See Bradley [17] for further discussion of this issue. On problems with the assignment of differentiated expert rights, see also Dietrich and List [3].

<sup>7</sup>Formally,  $\text{Pr}_i(XY|Z) = \text{Pr}_i(X|Z)\text{Pr}_i(Y|Z)$ .

<sup>8</sup>Formally, if, for all individuals  $i$ ,  $\text{Pr}_i(Z) > 0$  and  $\text{Pr}_i(XY|Z) = \text{Pr}_i(X|Z)\text{Pr}_i(Y|Z)$ , then also  $\text{Pr}(Z) > 0$  and  $\text{Pr}(XY|Z) = \text{Pr}(X|Z)\text{Pr}(Y|Z)$ .

It turns out, however, that Independence Preservation is violated by all linear pooling functions (unless some individual  $i$  gets maximal weight  $w_i = 1$ ) and thus by all non-dictatorial probability aggregation rules satisfying the above conditions of Event-wise Independence and Zero Preservation. This fact, proven in Genest and Wagner [7], can be illustrated using our earlier example. Suppose the aid agency consults a couple of experts in order to determine the risk of famine in a particular region and that both experts agree that famine is caused by a combination of drought (the event of rainfall  $R$  below some critical threshold) and political instability (the event of political conflict  $P$  above some critical threshold), which undermines local solutions to poor crop yields. Furthermore, they agree that these two factors are both causally and probabilistically independent, at least in the short term. But they disagree on the probability of drought and of political instability. Since neither speaks with greater authority than the other, the aid agency calculates its probabilities for these events by taking the average of the judgments of the two experts.

Let  $D$  and  $I$  respectively denote the occurrence of drought and political instability in the region and  $DI$  their concurrence.  $\text{Pr}_1$ ,  $\text{Pr}_2$ , and  $\text{Pr}$  are respectively the probability functions of Expert 1, Expert 2 and the aid agency. Since pooling happens by averaging, the aid agency will assign the following probabilities:

$$\begin{aligned}\text{Pr}(D) &= \frac{\text{Pr}_1(D) + \text{Pr}_2(D)}{2}, \quad \text{Pr}(I) = \frac{\text{Pr}_1(I) + \text{Pr}_2(I)}{2}, \\ \text{Pr}(DI) &= \frac{\text{Pr}_1(DI) + \text{Pr}_2(DI)}{2} = \frac{\text{Pr}_1(D)\text{Pr}_1(I) + \text{Pr}_2(D)\text{Pr}_2(I)}{2},\end{aligned}$$

where the last identity uses the experts' judgments that  $D$  and  $I$  are independent. These independence judgments are preserved if and only if  $\text{Pr}(DI) = \text{Pr}(D)\text{Pr}(I)$ , i.e., if and only if

$$\frac{\text{Pr}_1(D)\text{Pr}_1(I) + \text{Pr}_2(D)\text{Pr}_2(I)}{2} = \frac{\text{Pr}_1(D) + \text{Pr}_2(D)}{2} \times \frac{\text{Pr}_1(I) + \text{Pr}_2(I)}{2}.$$

By multiplying both sides of this equation by 4, developing the product on the right hand side, and simplifying, it follows that

$$\begin{aligned}\text{Pr}_1(D)\text{Pr}_1(I) + \text{Pr}_2(D)\text{Pr}_2(I) &= \text{Pr}_1(D)\text{Pr}_2(I) + \text{Pr}_2(D)\text{Pr}_1(I) \\ \Leftrightarrow \text{Pr}_1(D)(\text{Pr}_1(I) - \text{Pr}_2(I)) &= \text{Pr}_2(D)(\text{Pr}_1(I) - \text{Pr}_2(I)) \\ \Leftrightarrow (\text{Pr}_1(D) - \text{Pr}_2(D))(\text{Pr}_1(I) - \text{Pr}_2(I)) &= 0.\end{aligned}$$

The latter can hold only if  $\text{Pr}_1(D) = \text{Pr}_2(D)$  or  $\text{Pr}_1(I) = \text{Pr}_2(I)$ , i.e., if the experts agree on the probability of drought or of political instability – which is not the case by assumption. So equal-weight linear pooling violates Independence Preservation. Similar violations can be constructed for non-equal weights (unless one individual  $i$  gets maximal weight  $w_i = 1$ ).

While we have focused on linear pooling as a way of aggregating probability judgments, the difficulty with preserving causal insights at the aggregate level is a

very general one. Genest and Wagner [7] have shown that Independence Preservation is violated by many (linear or non-linear) probability aggregation rules, including geometric averaging, the most prominent alternative to linear averaging. Thus the difficulty of preserving causal knowledge is not an artifact of requiring Event-wise Independence (a condition violated for instance by geometric averaging).

Genest and Wagner [7] regard this finding as evidence that Independence Preservation is not a reasonable condition. We would not like to go so far. In our view, those unanimous independence judgments that are grounded in unanimous causal judgments about the world should *not* be overruled. We take Genest and Wagner’s impossibility finding not as a reason to abandon the goal of preserving judgments of independence, but as a reason to move to a two-stage approach that explicitly takes qualitative causal judgments into account.

### 3 Two-stage aggregation: the qualitative stage

Under our proposed two-stage approach to aggregation, qualitative causal judgments are aggregated first, and quantitative, probabilistic ones only subsequently. Moreover, the latter are aggregated in a way that differs from standard probability aggregation, namely in a way that is constrained by the qualitative causal judgments formed at the first stage. This two-stage approach will satisfy a version of Independence Preservation restricted to unanimously held *causal* independencies.

As before, let  $\mathbf{V} = \{V, W, \dots\}$  be a (finite) non-empty set of variables. In our example of the aid agency above,  $\mathbf{V}$  contains the variables  $R$  (rainfall),  $Y$  (crop yields),  $P$  (political conflict), and  $F$  (famine). How can we represent qualitative judgments on how the variables in  $\mathbf{V}$  are causally interrelated? Let us introduce a binary predicate symbol  $c$  to represent a causal relevance relation on  $\mathbf{V}$ , where, for any two variables  $V$  and  $W$  in  $\mathbf{V}$ , we write  $VcW$  to mean that  $V$  is causally relevant to  $W$ . In the case of the aid agency, an expert who thinks that rainfall is causally relevant to crop yield whereas political conflict is not would hold that  $RcY$  but not that  $PcY$ . A causal relevance relation  $c$  is called *acyclic* if, for any finite sequence  $V_1, V_2, \dots, V_k$  of variables in  $\mathbf{V}$ , it is not the case that

$$V_1cV_2, V_2cV_3, \dots, V_{k-1}cV_k \text{ and } V_kcV_1.^9$$

A causal relevance relation  $c$  induces a *directed graph*, whose vertices are the variables in  $\mathbf{V}$  and where, for any two variables  $V, W$  in  $\mathbf{V}$ , there is an edge from  $V$  in the direction of  $W$  if and only if  $VcW$ . This graph is a *directed acyclic graph (DAG)* if  $c$  is an acyclic relation.

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<sup>9</sup>Note that our definition of acyclicity also rules out cycles of length  $k = 1$ , i.e., we cannot have  $VcV$  for any variable  $V$ .

A *Bayesian network* is a DAG with associated conditional probabilities: each variable in the graph is endowed with a conditional probability distribution given its parents in the graph. In this section, however, we set this quantitative information aside and focus on qualitative features of the DAG alone. In particular, we investigate how a group of individuals can arrive at an aggregate judgment on what the causal relevance relation  $c$  between the variables in  $\mathbf{V}$  is.

Consider a group of  $n$  individuals, labelled  $1, 2, \dots, n$ , each of whom holds a particular judgment on the nature of the causal relevance relation between the variables in  $\mathbf{V}$ . We write  $c_i$  to denote the causal relevance relation according to individual  $i$ 's judgment. A combination of causal relevance relations across the  $n$  individuals is called a *profile* and denoted  $\langle c_1, c_2, \dots, c_n \rangle$ . A *causal judgment aggregation rule* is a function that assigns to each profile  $\langle c_1, c_2, \dots, c_n \rangle$  (in some domain of admissible profiles) a single aggregate causal relevance relation  $c$ .

To give some examples of causal judgment aggregation rules, consider the class of threshold rules. A *threshold rule*, with threshold  $k$  (where  $1 \leq k \leq n$ ), assigns to each profile  $\langle c_1, c_2, \dots, c_n \rangle$  the causal relevance relation  $c$  defined as follows: for any two variables  $V$  and  $W$  in  $\mathbf{V}$ ,

$$VcW \Leftrightarrow \text{at least } k \text{ individuals have } Vc_iW.$$

Examples of threshold rules are the *majority rule* ( $k = \frac{n+1}{2}$ ), the *union rule* ( $k = 1$ ) and the *intersection (or unanimity) rule* ( $k = n$ ).

Are these satisfactory causal judgment aggregation rules? It is easy to see that each of these three rules has a considerable defect. The majority and union rules fail to ensure acyclicity of the aggregate causal relevance relation, even when all individuals hold acyclic such relations. To see this, suppose the aid agency consults three experts, with the following individual judgments. They all agree that rainfall is causally relevant to crop yields, but they disagree on the causal relations between the other variables. Expert 1 thinks that crop yields are causally relevant to famine, which is causally relevant to political conflict. Expert 2 thinks that famine is causally relevant to political conflict, which is causally relevant to crop yields. Expert 3 thinks that political conflict is causally relevant to crop yields, which is causally relevant to famine. In consequence, the causal relevance relation generated by the majority rule violates acyclicity: the relation contains a cycle from crop yields to famines to political conflict to crop yields. It is obvious that the union rule has the same defect. The intersection (or unanimity) rule, by contrast, ensures acyclicity of the aggregate causal relevance relation, but may generate a sparse or even empty such relation, with few variables deemed causally relevant to any others, whenever there are disagreements between the experts.

Although threshold rules are particularly salient examples of causal judgment aggregation rules, they are by no means the only ones. So let us adopt an axiomatic

approach and look for rules satisfying certain conditions.

**UD** (*Universal Domain*) The causal judgment aggregation rule accepts as admissible any logically possible profile of acyclic causal relevance relations.

**AC** (*Acyclicity*) The aggregate causal relevance relation is always acyclic.

**UB** (*Unbiasedness*) For any two variables  $V$  and  $W$  in  $\mathbf{V}$ , the aggregate judgment on whether  $V$  is causally relevant to  $W$  depends only on individual judgments on whether  $V$  is causally relevant to  $W$ , and the aggregation rule is neutral between whether or not this is the case.<sup>10</sup>

**ND** (*Non-Dictatorship*) There does not exist a fixed individual such that, for every admissible profile of causal relevance relations, the aggregate causal relevance relation is the one held by that individual.

Although these conditions may seem natural at first sight, they are mutually inconsistent.

**Theorem 1** *If  $\mathbf{V}$  contains three or more variables, there exists no causal judgment aggregation rule satisfying UD, AC, UB and ND.*

We derive this result from an impossibility theorem by Dietrich and List [4] in the literature on judgment aggregation (for surveys, see List and Puppe [13] and List [12]). What are the possible escape routes from this impossibility result?

**The first route: relaxing universal domain.** We may use a causal judgment aggregation rule that accepts as admissible input not all logically possible profiles of acyclic causal relevance relations, but only those that meet an additional structural condition: namely profiles which, informally speaking, reflect a certain amount of cohesion – if not consensus – across different individuals’ causal judgments. The additional structural condition on profiles might be such that the majority rule, or perhaps some other threshold rule, never generates an aggregate causal relevance relation violating acyclicity. In this case, the majority rule or threshold rule in question could be employed on this restricted domain of admissible profiles. We consider two structural conditions of this kind.

*Temporal order restriction.* Suppose the individuals agree on the temporal order in which the events captured by the variables in  $\mathbf{V}$  occur. Suppose further they agree that a variable  $V$  can be causally relevant to another variable  $W$  only if  $V$  strictly precedes  $W$  in this temporal order. Call any profile of causal relevance relations that

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<sup>10</sup>Formally, for any  $V$  and  $W$  in  $\mathbf{V}$  and any admissible profiles  $\langle c_1, c_2, \dots, c_n \rangle$  and  $\langle c_1^*, c_2^*, \dots, c_n^* \rangle$ , if [for all  $i$ ,  $V c_i W$  if and only if not  $V c_i^* W$ ] then [ $V c W$  if and only if not  $V c^* W$ ]. This formal statement is slightly weaker than the informal one in the main text but implies the informal one together with UD and AC.

is consistent with *some* such agreement *temporal-order restricted*. Formally, a profile is *temporal-order restricted* if there exists *some* weak order of the variables in  $\mathbf{V}$  (a reflexive, transitive and connected binary relation on  $\mathbf{V}$ ) such that, for every pair of variables  $V$  and  $W$  in  $\mathbf{V}$ , if some individual judges  $V$  to be causally relevant to  $W$  (i.e., some  $i$  holds  $Vc_iW$ ) then  $V$  strictly precedes  $W$  in that order. For any such profile, the causal relevance relation generated by *any* threshold rule is acyclic, no matter how low or high the threshold is. The temporal constraint on what causal relevance judgments are deemed admissible guarantees the absence of any causal cycles at both the individual and aggregate levels.

*Unidimensional alignment.* Another structural condition on profiles that ensures acyclical causal judgments at the aggregate level – here under the majority rule (or any threshold rule with a higher threshold) – is *unidimensional alignment* (List [10]; for generalizations, see Dietrich and List [5]). A profile of causal relevance relations is called *unidimensionally aligned* if the individuals can be linearly ordered from left to right such that, for every pair of variables  $V$  and  $W$  in  $\mathbf{V}$ , the individuals who hold that  $V$  is causally relevant to  $W$  (i.e., the individuals  $i$  with  $Vc_iW$ ) lie either all to the left or all to the right of those who hold that  $V$  is not causally relevant to  $W$  (i.e., the individuals  $i$  who do not have  $Vc_iW$ ). For any unidimensionally aligned profile, the causal relevance relation generated by the majority rule is acyclic and coincides with the causal relevance relation held by the median individual with respect to the left-right ordering of the individuals (or, if the number of individuals is even, it coincides with the intersection of the causal relevance relations held by the two median individuals).

It is an empirical question whether a group of experts – either before or after a period of joint deliberation – exhibits sufficient agreement in their causal judgments to meet the condition of temporal order restriction or that of unidimensional alignment. The kind of temporal agreement required for temporal order restriction seems empirically plausible at least in some situations.

**The second route: relaxing acyclicity.** A logically possible way to avoid the impossibility result of Theorem 1 is to give up the requirement that the aggregate causal relevance relation be acyclic. This, however, would constitute a major departure from the consensus on the nature of causal relations, which are widely held to be acyclic (Pearl [16]).

**The third route: relaxing unbiasedness.** We may use a causal judgment aggregation rule which violates the condition of unbiasedness. Such a violation may take one of the following two forms.

*A neutrality violation.* There exist pairs of variables  $V$  and  $W$  in  $\mathbf{V}$  such that

the aggregation rule is not neutral between whether or not  $V$  is causally relevant to  $W$ . Examples of causal judgment aggregation rules violating neutrality are threshold rules with any threshold  $k$  different from simple majority. It can be shown that a threshold rule is guaranteed to generate an acyclic causal relevance relation if and only if the threshold  $k$  exceeds  $\frac{m-1}{m}n$ , where  $m$  is the number of variables in  $\mathbf{V}$ .<sup>11</sup> If the set of variables  $\mathbf{V}$  is infinite, only the intersection (or unanimity) rule guarantees acyclicity at the aggregate level, but if  $\mathbf{V}$  is finite, then a supermajority rule with a suitably high threshold is sufficient. A problem with this approach, as noted above, is that it may lead to sparse or even empty aggregate causal relevance relations unless the disagreement between experts is limited.

*An independence violation.* In this case, there exist pairs of variables  $V$  and  $W$  in  $\mathbf{V}$  such that the aggregate judgment on whether  $V$  is causally relevant to  $W$  depends not only on individual judgments on whether  $V$  is causally relevant to  $W$  but also on individual judgments involving other variables. Examples of causal judgment aggregation rules violating independence are *sequential priority rules* (List [11]). Here the different possible pairs of variables are considered one by one in a given order (which may be chosen, for example, by some criterion of epistemic priority). On each pair of variables  $V, W$ , the aggregate judgment is then determined as follows:

- (i) If the question of whether  $V$  is causally relevant to  $W$  is constrained (in light of the acyclicity requirement) by the aggregate judgments on pairs of variables considered earlier in the given order, then the aggregate judgment on  $V$ 's causal relevance to  $W$  is dictated by those earlier constraints.
- (ii) If it is not constrained in this way, then the aggregate judgment on  $V$ 's causal relevance to  $W$  is made by applying some voting method, such as majority voting, to the individual judgments on  $V$  vis-à-vis  $W$ .

This approach guarantees acyclicity of the aggregate causal relevance relation, but at the expense of “path-dependence”: the order in which causal judgments are made on different pairs of variables may determine what the aggregate causal relevance relation will look like. An agenda setter on a committee of experts may strategically exploit this feature of the causal judgment aggregation rule by proposing an order of priority among different pairs of variables that is likely to give rise to aggregate causal judgments that the agenda setter wants the committee to make.

**The fourth route: relaxing non-dictatorship.** A final way to avoid the impossibility result of Theorem 1 is to allow the aggregate causal relevance relation to be determined by an antecedently fixed individual: a “dictator”. But since we are normally interested in the information contained in the causal judgments of more than

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<sup>11</sup>This can be derived from a characterization of consistent (but possibly incomplete) quota rules in judgment aggregation in Dietrich and List [2].

one individual, this is not generally an attractive solution to our aggregation problem. Sometimes, however, it may be an acceptable compromise to appoint a trusted expert as the “dictator” for arriving at qualitative causal judgments – in the form of a DAG – while continuing “democratically” when it comes to determining the associated quantitative probability information at the second stage of our two-stage approach.

Which of the different possible escape routes from the impossibility result of Theorem 1 is compelling depends on details of the decision problem at hand, the nature of the disagreements between the experts, the level of trust we place in them, and so on. In the next section, we assume that through one of these routes – excluding that of relaxing acyclicity – a “consensus” on a causal relevance relation and thereby on a DAG has been achieved, and we turn to the question of how the associated conditional probabilities can be determined.

## 4 Preliminaries to the quantitative stage

We have analysed how a group can arrive at an aggregate judgment on the qualitative causal relations between variables. We now assume that such an aggregate causal judgment has been reached through one of the routes just discussed and suppose that the group seeks to make an aggregate probability judgment (about the variables taking various values) that is compatible with the given aggregate causal judgment.

In its most general form – ignoring for the moment the causal judgment – a probability judgment can be represented by a joint probability function over the variables in  $\mathbf{V}$ . For simplicity, we assume that each variable can take finitely, or countably infinitely, many possible values. For example, we may distinguish between a particular number of possible levels of conflict. Let us label the variables  $V_1, \dots, V_m$ . A joint probability function  $\Pr$  assigns a probability  $\Pr(v_1, \dots, v_m) \geq 0$  to each combination  $(v_1, \dots, v_m)$  of values of these variables, where the sum of the probabilities is 1.

The joint probability  $\Pr(v_1, \dots, v_m)$  can be factorised into the product of conditional probabilities:<sup>12</sup>

$$\begin{aligned} \Pr(v_1, \dots, v_m) &= \Pr(v_1) \Pr(v_2|v_1) \Pr(v_3|v_2, v_1) \cdots \Pr(v_m|v_{m-1}, \dots, v_1) \\ &= \prod_{j=1}^m \Pr(v_j|v_1, \dots, v_{j-1}). \end{aligned} \tag{1}$$

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<sup>12</sup>In this expression, the conditional probability  $\Pr(v_j|v_1, \dots, v_{j-1})$  can be derived from the joint probability function  $\Pr$  via the formula  $\Pr(v_j|v_1, \dots, v_{j-1}) = \frac{\Pr(v_1, \dots, v_j)}{\Pr(v_1, \dots, v_{j-1})}$  (where  $\Pr(v_1, \dots, v_j)$  and  $\Pr(v_1, \dots, v_{j-1})$  are marginal probabilities derived from  $\Pr$ ), *provided* that  $\Pr(v_1, \dots, v_{j-1}) \neq 0$ . If  $\Pr(v_1, \dots, v_{j-1}) = 0$ ,  $\Pr(v_j|v_1, \dots, v_{j-1})$  can be viewed either as undefined or as a primitive not derived from the function  $\Pr$ . Under both interpretations, the factorisation (1) is still possible even if some  $\Pr(v_1, \dots, v_{j-1})$  is zero *whatever* value is substituted for  $\Pr(v_j|v_1, \dots, v_{j-1})$  (because some other factor on the right-hand side of (1) will be zero, as will be the left-hand side of (1)).

In our famine example, where  $V_1, V_2, V_3, V_4$  are the levels of rainfall, crop yield, political conflict and famine, we have

$$P(v_1, v_2, v_3, v_4) = P(v_1)P(v_2|v_1)P(v_3|v_1, v_2)P(v_4|v_1, v_2, v_3).$$

When is the probability judgment expressed by  $\Pr$  *compatible* with a given causal judgment? Recall that a causal judgment takes the form of a particular directed acyclic graph (DAG) over the variables  $V_1, \dots, V_m$ , with an arrow from  $V_j$  to  $V_k$  just in case  $V_j$  is considered causally relevant to  $V_k$  ( $V_j c V_k$ ). For any variable  $V_j$ , we write  $PA(V_j)$  to denote the list of  $V_j$ 's parent variables in the graph, and we write  $pa(V_j)$  to denote any list of values of these parent variables.<sup>13</sup>

For instance, suppose that the consensus DAG in our famine example is as shown in Figure 2: no variable is causally relevant to rainfall ( $V_1$ ); only rainfall ( $V_1$ ) is causally relevant to crop yield ( $V_2$ ); only crop yield ( $V_2$ ) is causally relevant to political conflict ( $V_3$ ); but both crop yield ( $V_2$ ) and political conflict ( $V_3$ ) are causally relevant to famine ( $V_4$ ). Then  $PA(V_1)$  contains no variable,  $PA(V_2)$  contains precisely  $V_1$ ,  $PA(V_3)$  contains precisely  $V_2$ , and  $PA(V_4)$  contains both  $V_2$  and  $V_3$ .

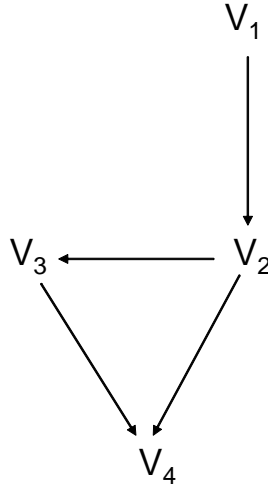


Figure 2: An illustrative aggregate causal judgment in the famine example

Without loss of generality, suppose the variables  $V_1, \dots, V_m$  are labelled such that those with no parent come first, those with a parent but no grandparent come next, those with a grandparent but no great-grandparent come thereafter, and so on. If the original labelling  $V_1, \dots, V_m$  does not have this property, we can simply relabel the variables appropriately and replace the factorisation (1) by one using the new labelling. So the parents of any variable  $V_j$  come before  $V_j$ .<sup>14</sup> But of course not all

<sup>13</sup>So  $pa(V_i)$  is any instantiation of  $PA(V_j)$ .

<sup>14</sup>Formally,  $PA(V_j)$  is a sublist of  $(V_1, \dots, V_{j-1})$ .

of  $V_1, \dots, V_{j-1}$  need to be causally relevant to  $V_j$ . For instance, in our famine example  $V_2$  but not  $V_1$  is (directly) causally relevant to  $V_3$ . As causally irrelevant variables should have no effect on  $V_j$ , the conditional probability  $\Pr(v_j|v_1, \dots, v_{j-1})$  should be insensitive to the non-parental values among  $v_1, \dots, v_{j-1}$ . In other words, it should be sensitive only to the sublist  $pa(V_j)$  of  $v_1, \dots, v_{j-1}$ . Formally,

$$\Pr(v_j|v_1, \dots, v_{j-1}) = \Pr(v_j|pa(V_j)). \quad (2)$$

We say that the probability judgment  $\Pr$  is *compatible* with the given aggregate causal judgment if identity (2) holds for every variable  $V_j$  and every combination of values  $v_1, \dots, v_j$  with  $\Pr(v_1, \dots, v_{j-1}) \neq 0$ .<sup>15</sup> The joint probability (1) then reduces to

$$\Pr(v_1, \dots, v_m) = \prod_{j=1}^m \Pr(v_j|pa(V_j)). \quad (3)$$

For instance, in our famine example,

$$P(v_1, v_2, v_3, v_4) = P(v_1)P(v_2|v_1)P(v_3|v_2)P(v_4|v_2, v_3).$$

## 5 Two-stage aggregation: the quantitative stage

As we seek to reach an aggregate probability judgment that is compatible with the aggregate causal judgment, the probability function  $\Pr$  should satisfy the decomposition (3). This requirement is usually violated by standard, one-stage probability aggregation, where the individual probability functions

$$\Pr_1(v_1, \dots, v_n), \dots, \Pr_n(v_1, \dots, v_n) \quad (4)$$

are directly merged into an aggregate probability function  $\Pr(v_1, \dots, v_n)$ . On our proposed two-stage approach, by contrast,  $\Pr$  is explicitly constructed so as to meet the necessary decomposition requirement.

Let the aggregate causal relevance relation (the “consensus” DAG) be given, and consider the decomposition constraint (3) relative to that relation. The quantitative stage of our approach now consists in

- (i) determining each factor of the decomposition,  $\Pr(v_j|pa(V_j))$ , through separate probability aggregation, and
- (ii) computing the joint probability function  $\Pr(v_1, \dots, v_m)$  as the product of these separately determined factors.

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<sup>15</sup>There are different equivalent ways to define “compatibility” of  $\Pr$  with the DAG. While we have defined compatibility by the *ordered Markov condition*, an equivalent definition uses the *parental Markov condition*: any variable is independent of its non-descendants given its parents. A third definition (chosen by Pearl) is given in terms of the validity of the decomposition (4). For the equivalence of these definitions, see Theorems 1.2.6 and 1.2.7 in Pearl [16].

More formally, for *every* variable  $V_j$  in  $\mathbf{V}$  and *every* combination  $pa(V_j)$  of parental values, we merge the individual conditional probability functions

$$\Pr_1(v_j|pa(V_j)), \dots, \Pr_n(v_j|pa(V_j)) \quad (5)$$

into an aggregate conditional probability function  $\Pr(v_j|pa(V_j))$ . These separate aggregation exercises can each be performed, for example, by linear or geometric pooling. In our famine example, this involves

$$\begin{array}{lll} \text{merging } \Pr_1(v_1), \dots, \Pr_n(v_1) & & \text{into } \Pr(v_1), \\ \text{for any fixed } v_2, \quad \text{merging } \Pr_1(v_2|v_1), \dots, \Pr_n(v_2|v_1) & & \text{into } \Pr(v_2|v_1), \\ \text{for any fixed } v_3, \quad \text{merging } \Pr_1(v_3|v_2), \dots, \Pr_n(v_3|v_2) & & \text{into } \Pr(v_3|v_2), \\ \text{for any fixed } v_2, v_3, \quad \text{merging } \Pr_1(v_4|v_2, v_3), \dots, \Pr_n(v_4|v_2, v_3) & & \text{into } \Pr(v_4|v_2, v_3). \end{array} \quad (6)$$

The present approach has several distinctive properties, to which we now turn.

**Compatibility with causal judgments.** The aggregate probability function  $\Pr$ , given by (3), is automatically compatible with the aggregate causal relevance relation, represented by the appropriate DAG. In particular,  $\Pr$  respects the causal Markov condition: any variable  $V_j$  is probabilistically independent of all its causal non-descendants given its causal parents (recall footnote 15). In our famine example,  $\Pr$  makes political conflict independent of rainfall conditional on crop yield,<sup>16</sup> and famine independent of rainfall conditional on crop yield and political conflict.<sup>17</sup> The causally motivated conditional independencies are thus respected, whereas other conditional independencies may or may not arise. By contrast, standard one-stage probability aggregation does not generally produce an aggregate probability judgment that is consistent with any prior judgments of causal relevance.

**Preservation of causal (conditional) independencies.** What about the preservation of unanimously held independencies between variables (both conditional and unconditional ones)? Suppose, for example, that all individuals consider variables  $V_j$  and  $V_k$  probabilistically independent given  $V_l$ .<sup>18</sup> Does the aggregate probability judgment preserve this conditional independence? As we have seen, for standard probability aggregation methods the answer is usually negative. Under our approach, by contrast, *causal* conditional independencies are preserved. To see why, suppose all individuals judge  $V_j$  and  $V_k$  to be probabilistically independent given  $V_l$  because of a unanimous agreement that  $V_j$ 's only causal parent is  $V_l$  and that  $V_k$  is not a causal descendant of  $V_j$ . Then the aggregate probability judgment respects this independence: according to  $\Pr$ ,  $V_j$  and  $V_k$  are also probabilistically independent given  $V_l$ .<sup>19</sup>

<sup>16</sup>Formally,  $\Pr(v_1, v_3|v_2) = \Pr(v_1|v_2) \Pr(v_3|v_2)$ .

<sup>17</sup>Formally,  $\Pr(v_1, v_4|v_2, v_3) = \Pr(v_1|v_2, v_3) \Pr(v_4|v_2, v_3)$ .

<sup>18</sup>Formally,  $\Pr_i(v_j, v_k|v_l) = \Pr_i(v_j|v_l) \Pr_i(v_k|v_l)$ .

<sup>19</sup>Formally,  $\Pr(v_j, v_k|v_l) = \Pr(v_j|v_l) \Pr(v_k|v_l)$ .

The reason is that, so long as a “reasonable” causal judgment aggregation rule is used at the first stage of our two-stage process, we will have arrived at an aggregate causal relevance relation that reflects the unanimous opinion on the causal relations between  $V_j, V_k, V_l$ ; the second stage then leads to a probability function that is compatible with this aggregate causal relevance relation.

**Variable expert weights** In contrast to one-stage linear or geometric pooling of probabilities, our approach is compatible with the assignment of different weights to different experts’ judgments so as to reflect their different levels of competence on the relevant issues. Once the consensus DAG for the causes of famine is given, for instance, greatest weight can be assigned to the climatologist’s judgment in the aggregate probability for rainfall ( $\Pr(v_1)$ ), to the agriculturalist’s judgment in the aggregate conditional probability for crop yield, given a level of rainfall ( $\Pr(v_2|v_1)$ ), and to the political scientist’s judgment for the aggregate conditional probability for political conflict, given crop yields ( $\Pr(v_3|v_2)$ ). In the limit, an aggregate judgment on the probability of famine might be constructed using only the consensus DAG and the judgments of the relevant expert on each variable. But as the literature on epistemic democracy shows, there can be advantages to consulting a range of opinions provided that all who are consulted are sufficiently competent. Instead, the two-stage method can be used to optimise the balance between competence and diversity of opinion by suitable assignment of weights in the aggregation of probabilities for each variable.

**Complexity reduction.** Our two-stage approach subdivides an  $m$ -dimensional probability aggregation problem into several one-dimensional ones. Rather than aggregating joint probability functions over the vector  $V_1, \dots, V_m$  (of the form (4)), we aggregate conditional probability functions of a single variable  $V_j$  (of the form (5)). But we face several such aggregation problems, namely one for each variable  $V_j$  and each fixed combination of parent values  $pa_j(V_j)$ . This is less demanding on the side of individual inputs, as long as the aggregate DAG is not too rich in causal connections. To illustrate this complexity reduction, consider our famine example again, and suppose for simplicity that each variable can take only two values, i.e., there are only two levels of rainfall, two levels of crop yield, and so on. If we were to aggregate the joint probability functions  $\Pr_i(v_1, v_2, v_3, v_4)$  directly, each individual would have to submit  $2^4 - 1 = 15$  probability values (there are  $2^4$  possible combinations of values  $(v_1, v_2, v_3, v_4)$ , but once the probabilities of  $2^4 - 1$  of them are specified, the remaining probability is given by one minus the sum of the rest). Specifying any one of these 15 probabilities is hard in practice: what, for example, is the probability of a combination of high rainfall *and* low crop yield *and* low political conflict *and* high famine? Under our approach, by contrast, each individual has to submit only probabilities or conditional probabilities of *singular* events, like the probability of high rainfall or the

conditional probability of high crop yield given low rainfall. The number of required probabilities is smaller than 15 in our example. Using (6), we can see that it equals

$$\begin{aligned}
& \sum_{j=1}^4 \text{“number of possible values of } V_j \text{ minus 1”} \\
& \quad \times \text{“number of possible parent values } pa(V_j)\text{”} \\
= & (2 - 1) \times 1 + (2 - 1) \times 2 + (2 - 1) \times 2 + (2 - 1) \times 2^2 \\
= & 1 + 2 + 2 + 4 = 9.
\end{aligned}$$

**Types of informational input.** Our approach not only reduces the complexity of the aggregation problem; it also uses a different informational input, compared to one-stage probability aggregation. First, we use the additional information of the individuals’ qualitative causal judgments – the information aggregated at the first stage of our two-stage process. Second, an interesting question arises about the nature of the probabilistic input used at the second stage. Consider a variable  $V_j$  with parents  $PA(V_j)$  in the aggregate causal relevance relation (DAG). Since that relation is the result of the aggregation of individual causal relevance relations, some individuals may not agree that the variables listed in  $PA(V_j)$  are the correct causal parents of  $V_j$ . They may think instead that not all of these variables are causally relevant to  $V_j$  or that some other variables are relevant, despite not being included in  $PA(V_j)$ . But then, what does such an individual’s conditional probability  $\Pr_i(v_j|pa(V_j))$  – the informational input at the second stage – represent? In our famine example, what is someone supposed to answer to the question “how probable is high political conflict given low crop yield?” if he or she actually thinks that famine rather than crop yield is causally relevant to political conflict?

There are (at least) two possible interpretations of an individual’s conditional probabilities here: an *evidential* and a *causal* one. To give an informal example of the difference, suppose for a moment that, according to individual  $i$ ’s qualitative causal judgment, the variables in  $PA(V_j)$  are not *causally* relevant to  $V_j$  but nonetheless *probabilistically correlated* with  $V_j$ . Then, if  $\Pr_i(v_j|pa(V_j))$  represents an evidential conditional probability, its value is sensitive to  $pa(V_j)$  (by probabilistic dependence), whereas if it is understood as a causal conditional probability, its value does not depend on  $pa(V_j)$  (by causal independence). More generally, an evidential conditional probability represents an agent’s belief, given a particular *evidential supposition* (here the supposition that the values of the variables in  $PA(V_j)$  are  $pa(V_j)$ ). By contrast, a causal conditional probability represents an agent’s belief, given a particular *counterfactual supposition* (its content again being that the values of the variables in  $PA(V_j)$  are  $pa(V_j)$ ). This causal conditional probability can be understood as resulting from a hypothetical external intervention into the system which sets the values of the variables  $PA(V_j)$  to  $pa(V_j)$ . The two kinds of conditional probability take the same

value if  $PA(V_j)$  happens to consist precisely of the correct causal parents *according to individual  $i$ 's qualitative causal judgment*, but may differ in general.

Formally, in the evidential case,  $\Pr_i(v_j|pa(V_j))$  is a standard conditional probability, which can be derived from individual  $i$ 's joint probability function over the variables using Bayes's rule.<sup>20</sup> In the causal case,  $\Pr_i(v_j|pa(V_j))$  can be calculated as follows (and is sometimes denoted  $\Pr_i(v_j|pa(V_j))$  or  $\Pr_i(v_j \setminus pa(V_j))$  to mark the difference; see also Pearl [16]).

- (i) Modify individual  $i$ 's causal relevance relation by deleting relevance links from any variable to any of the variables in  $PA(V_j)$ . So, the variables in  $PA(V_j)$  have no parents left (intuitively, they are set by an external intervention).
- (ii) Modify the probability assignment to the variables  $PA(V_j)$  by letting them take the values  $pa(V_j)$  with probability one (unconditionally, since these variables no longer have any parents).
- (iii) Relative to this new "post-intervention" Bayesian network, compute the probability that  $V_j$  takes the value  $v_j$  in the usual way. This probability then coincides with the causally understood conditional probability  $\Pr(v_j|pa(V_j))$  of the initial Bayesian network.

Apart from the evidential and causal interpretations of the individuals' conditional probabilities submitted at the second stage of our two-stage process, we can imagine a third possible interpretation in which individuals are asked to entertain the hypothesis that the aggregate causal relevance relation is correct and to express conditional probabilities based on this hypothesis. Here, however, it is less clear how and whether  $\Pr_i(v_j|pa(V_j))$  can be derived from the individuals' Bayesian networks.

## 6 A challenge

The first stage of our two-stage approach restricts the second by requiring the aggregate probability function to display certain conditional independencies mandated by the aggregate causal relevance relation. Roughly, the fewer causal links are accepted at the first stage, the more probabilistic independencies are enforced at the second stage. In the extreme case in which *no* variable is deemed causally relevant to any other variable, the second stage produces an aggregate probability judgment according to which every variable is probabilistically independent of every other. Accepting few causal connections has the advantage of reducing the complexity of the probability aggregation problem at the second stage but the potential disadvantage of over-restricting the admissible probability assignments. This restriction is problematic when the sparse set of accepted causal links between variables is not a result

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<sup>20</sup>Provided that  $\Pr(pa(V_j)) \neq 0$ .

of the individuals believing in sparse causal links but a result of a causal judgment aggregation rule setting a high threshold for the acceptance of causal links.

We are thus faced with a trade-off between the goal of reducing the complexity of the probability aggregation problem (achieved via a high threshold for accepting causal links between variables) and the goal of representing causal effects between variables whenever there are such effects (achieved via a low threshold for accepting causal links). We have argued that a high threshold for accepting causal links may help to prevent a cyclical aggregate causal relevance relation, whereas in other situations, particularly if the variables can be put into a temporal order, even a low threshold (perhaps lower than the majority threshold) guarantees acyclicity. We leave it as a challenge for future research to come up with causal judgment aggregation rules that perform well on both aspects of this trade-off: being neither too permissive nor too restrictive in accepting causal links while avoiding cyclical causal judgments.

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